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On the Detection of Black Holes

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It is shown that in principle the gravitational lens effect may lead to significant light variations when a collapsed object such as a black hole passes between the observer and a normal star. Light curves characteristic of such an event are computed, and the possibility of observing such an event is discussed.

Key words: black holes — lens effect — observations

I. Introduction

The General Theory of Relativity coupled with reasonable assumptions concerning the equation of state for matter with over-nuclear density points to the possible existence of "black holes", that is to say, to bodies which have undergone complete gravitational collapse, have zero luminosity and have radii determined by their masses (Oppenheimer *et al.*, 1939). The radii are given by $r = 2GM/c^2$ where M is the mass of the body, G the gravitational constant and c the speed of light. This value of r is very small indeed: it would be about 3 km for a mass equal to that of the sun.

With zero luminosity and such small dimensions, black holes can be detected only through the effects they have on nearby matter and radiation. Specifically, the gravitational field of a black hole will alter the trajectory of neighboring bodies and light. However, any dark body acts in this respect in an identical way provided the neighboring bodies or light are at appropriate distances. The recent discussion by Cameron (1971) as to the nature of the invisible companion of ε Aurigae, demonstrates the need for a characteristic criterion distinguishing black hole behaviour. In the absence of such a criterion, one cannot avoid the natural temptation to associate little-understood phenomena with black holes (Hawking, 1971). The accretion of neighboring matter by a black hole could produce X-radiation (Zeldovich *et al.*, 1964) but this radiation could also be produced in other processes not related to black holes. In this respect, it would be helpful to be able to determine a small enough upper limit to the dimension of an invisible object to permit distinguishing a black hole from the smallest known "normal" stars.

The lens effect arising from the bending of light in the neighborhood of a massive body could be useful since the strength of the effect depends on the proximity of the light trajectory to the body. It has recently been stated by Trimble and Thorne (1969) that the gravitational lens effect produced by a collapsed star or neutron star would be too small to be measured. We do not agree with this conclusion, and in what follows we present the results of an analysis that demonstrates the possibility of detecting black holes through their lens effect with particular reference to binary systems in which one component is a normal star while its companion is a collapsed star.

II. The Gravitational Lens Effect

Consider the situation in which a small, dense, dark possibly collapsed star (the "lens-star") is seen by an observer to be projected onto the disk of a normal star. The lens effect on an infinitesimal part of the normal star located at a projected radial distance r from the lens-star is given by

$$L/L_0 = \left(1 + \frac{8Mx}{r^2}\right)\left(1 + \frac{16Mx}{r^2}\right)^{-1/2} \qquad (1)$$

where L is the luminosity as affected by the lens effect, L_0 is the luminosity in the absence of the lens effect, M is the mass of the lens-star and x is the separation of the two stars. When the two stars are perfectly aligned, the integrated effect for the whole star is found by integrating Eq. (1) from $r = 0$ to $r = R$, the radius of the star, and is given by

$$L/L_0 = \left(1 + \frac{16 M x}{R^2}\right)^{1/2} \quad (2)$$

where L and L_0 now refer to the entire star. We are not giving the details of the derivation for the following reasons: Eq. (1) can be derived directly from equation 18 of the paper by Refsdal (1964) by going over from his angular parameters to ours and by taking into consideration that the distance between the two stars is negligible compared with their distance from the observer on Earth; Eq. (2) apart from being a direct consequence of Eq. (1), is given, without derivation, in the paper by Trimble and Thorne (1969). Expressing mass and radius in solar units and the separation in astronomical units, Eq. (2) becomes

$$\frac{L}{L_0} = \left(1 + 7.34 \times 10^{-3} \frac{M x}{R^2}\right)^{1/2}. \quad (3)$$

It is easily shown that a wide range of *reasonable* values for R and x will give a value for L/L_0 which differs significantly from unity. For example, let the mass of the collapsed star be $M = 30$ (a value which we adopt later as being typical for a black hole), the radius of the normal star $R = 1$, and the separation $x = 50$, which is near the peak of the frequency distribution of semi-major axes of normal binaries Heintz (1969). Then we have, $L/L_0 = 3.46$, an increase in the flux during a central eclipse of more than two-hundred percent!

Consider the case of ε Aurigae, which Cameron (1971) has recently suggested may have a black hole component. At present ($R \cong 1000$, $x \cong 35$) the gravitational lens effect would be negligible. However, if the visible component has evolved from a point initially on the upper-main sequence ($R \cong 10$, say), and if at this early stage its companion was already collapsed, then a distant observer in the orbit plane would have seen a light amplification given by $L/L_0 = 1.03$, an increase of three percent, or three one-hundredths of a magnitude which is readily detected.

III. Theoretical Light Curves

Having shown that the passage of a massive, collapsed star between an observer and a normal star can give rise to detectable light changes, and recognizing that such an event is most likely to occur in a binary system, we now examine the light variation to be expected during an eclipse in such a system. The lens effect when the eclipse is non-central, i. e., when the two stars and the observer are not aligned, can be calculated according to the following expression

$$L/L_0 = \frac{1}{\pi R^2} \int r^2 \, d\alpha \left(1 + \frac{16 M x}{r^2}\right)^{1/2} \quad (4)$$

in which r and α are polar coordinates over the surface of the star which is treated as a disk. The origin of the polar coordinates is the intersection between the stellar disk and the line joining the Earth to the centre of the lens-star. If u is the distance between the origin of the polar coordinates so defined and the centre of the visible star, we have

$$R^2 = u^2 + r^2 - 2ur \cos\alpha \quad (5)$$

from which $d\alpha$ can be found in terms of r and dr leading to the following expression

$$\frac{L}{L_0} = \frac{1}{\pi R^2} \int_{R-u}^{R+u} \frac{(R^2 - u^2 + r^2)\, dr (r^2 + 16 M x)^{1/2}}{(4 r^2 u^2 - (R^2 - u^2 - r^2)^2)^{1/2}}. \quad (6)$$

Let us perform the transformation $r^2 = 4\,Ruz + (R-u)^2$, and set $a = R/u$ and $b = 16 Mx/R^2$. We then have

$$\frac{L}{L_0} = \frac{1}{a\pi} \int_0^1 \frac{2z + a - 1}{(z - z^2)^{1/2}} \left(\frac{4az + (a-1)^2 + a^2 b}{4az + (a-1)^2}\right)^{1/2} dz. \quad (7)$$

This integral has been calculated for a range of values of a and b sufficiently wide to give the variation in light intensity throughout an entire eclipse and for different orientations of the orbit plane. Some representative results are illustrated in Fig. 1 and Fig. 2. In each case, the solid curve illustrates the light variation for an inclination of the orbit plane to the plane of the sky of 90°, whereas the dashed curve applies to the case where the inclination is less and such that the closest projected approach of the centres of the two stars is $R/2$. We use a normalized unit of time, T, which is the time during which the lens-star remains in the cone defined by the edge of the normal star and the observer, i. e., the normal duration of an eclipse.

For the case of an observer in the orbit plane, the light amplification factor varies very slowly during the time the lens-star remains within the cone

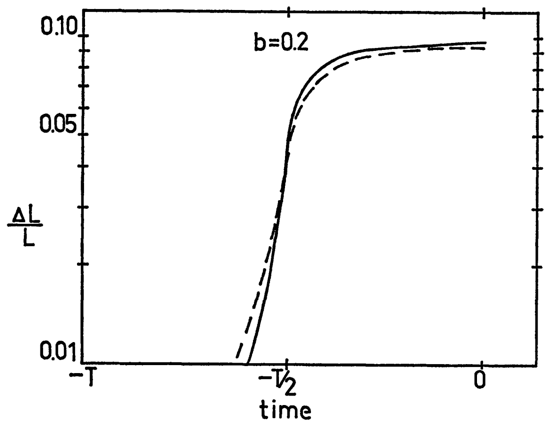

Fig. 1. Time variation in apparent luminosity, expressed as $\Delta L/L_0$, from one characteristic time unit before mid-eclipse to mid-eclipse for the case $\frac{16Mx}{R^2} = 0.2$. Solid curve: 90° inclination. Dashed curve: closest approach of projected centres of stars is $R/2$

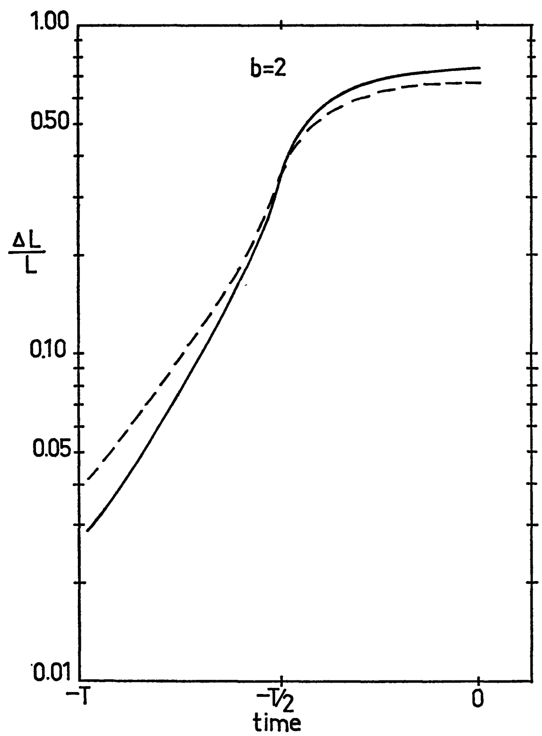

Fig. 2. Same as Fig. 1 but for the case $\frac{16Mx}{R^2} = 2.0$.

defined by the normal star and the observer. However, this factor drops very sharply to one-half its maximum value as the lens-star reaches the edge of the cone ($a = 1$ or $u = R$). When the lens-star is outside the cone the relative change in brightness is strongly dependent on the value of Mx/R^2. If, for example, $Mx/R^2 = 0.0168$ the additional luminosity at $u = 1.2R$ is one-eighth that for $u = 0$ (stars perfectly aligned with the observer). However, for $Mx/R^2 = 0.0687$ the effect at $u = 1.2R$ is greater than one-fifth that for $u = 0$. The point to be emphasized is that, for the eclipse of a normal star by a collapsed star, the variation in brightness as a function of time is characteristically different from that obtained in the case of an eclipse by a large body, and gives rise to a light variation which can be distinguished, in most cases, from that of an intrinsically variable star.

In order that the lens effect will occur as calculated, the eclipsing star must not have too large a radius since it would otherwise intercept light rays normally received from the visible star by the observer. It can easily be shown that, because of the bending of light, no ray going from the star to the observer will approach the lens star to a coordinate distance smaller than $r_0 = \frac{4Mx}{R}$ in the case where the two stars and the observer are perfectly aligned. Therefore, any star with coordinate radius sensibly greater than this value will not produce the calculated effect. A better indication of the restriction imposed by this result is possible if we rewrite this limiting expression in the form $r_0/R = 4Mx/R^2$. This quantity varies, in our case, from 2.5×10^{-2} to more than 100. We see, therefore, that there are cases in which non-collapsed stars could also produce the lens effect. In such a case the detection of the effect might allow for a better check of the gravitational bending of light than is possible during a solar eclipse. As a test of the General Theory of Relativity, the measurement of the bending of light near the sun during a solar eclipse still leaves much to be desired in the way of accuracy (Adler *et al.*, 1965). In the case of the lens effect one is not faced with the difficult task of measuring the small displacements of stellar images on a photographic plate: it would be enough to find a time variation of brightness in agreement with that calculated according to the lens effect in order to have a good check of the bending effect.

When the two stars are luminous with luminosity A_0 and B_0 the total normal luminosity will be $L_0 = A_0 + B_0$. At the time of the eclipse we will have $L = A + B_0$, and the apparent lens effect will be given by

$$\frac{L}{L_0} = \frac{A + B_0}{A_0 + B_0} = 1 + \frac{A - A_0}{A_0 + B_0} \quad (8)$$

and is smaller than the "true" lens effect. The discrepancy between the two values of L/L_0 — the one deduced from the apparent lens effect, the other deduced from the time variation — may allow one to distinguish between the case of two luminous stars and the case in which one of the stars is a dark, collapsed star.

IV. Observational Considerations

In this section we present the results of very approximate calculations relating to the possibility of observing the type of eclipse phenomenon described above, and, therefore, the possibility of detecting black holes by virtue of their presence in eclipsing binary systems. These results are necessarily very uncertain for a number of reasons: we cannot say with complete certainty whether black holes do, or even can, exist; we cannot, therefore, say much about their mass distribution, relative frequency in the neighborhood of the sun, frequency distribution of orbital parameters if they exist in binary systems, etc. Many of these quantities are uncertain even among "normal" stars.

One can easily show that in a binary system in which one component is very much smaller than the other, the probability of observing an eclipse (small star in front) from a randomly selected direction in space and at a randomly selected moment in time is approximately

$$2 \times 10^{-6} \frac{R^2}{x^2}$$

where R is the radius of the normal star in solar units, and x is their separation (we assume circular orbits) in astronomical units. We assume that among all stellar systems — single, binary, multiple — one-percent are binaries in which one component is a normal star and the other is a collapsed star. We further assume that among these "collapsar binaries" the frequency distribution of semi-major axes is identical to that for binaries consisting of two normal stars as given by Heintz (1969). We assume, as indicated earlier, that the typical collapsed star, or black hole, has a mass $M = 30\ \odot$, and that the typical companion to a black hole has a radius and mass each of three solar units. We further limit the discussion to cases in which the light amplification amounts to 5%, or more. It follows that we are considering systems with periods greater than about $1^1/_2$ years, and, therefore, in order to avoid missing an eclipse event, the frequency of observation must be at least one per day.

On the basis of the above, we have calculated that among the approximately 10^8 stars brighter than apparent visual magnitude 15, the expected number of black hole eclipses per day is 0.06. Or, if all the stars to a limiting apparent visual magnitude of 15 were observed daily we could expect to detect one collapsar binary eclipse every two to three weeks. These expectation values can be scaled up or down in proportion to the number of stars observed and the assumed frequency of black hole binaries. A single observer equipped with a telescope which covers approximately 30 square degrees of the sky in a single exposure, and photographing ten "average" regions of the sky per night to the same magnitude limit, could expect to continue for one to two thousand nights before recording an event. To say that one black hole eclipse event could be observed in a given time interval is not, of course, the same as saying that one black hole could be positively identified during that time interval. Having discovered a star which is abnormally bright at some moment, the time-consuming task of making additional observations in order to establish the character of the variation remains.

V. Conclusions

We have tried to demonstrate in this paper that in the case of an eclipse of a normal star by a dark, collapsed body, the gravitational lens effect may appreciably alter the apparent brightness of the star, and, furthermore, the light variation will have a very characteristic form. From the shape of the light curve, it should be possible to distinguish a black hole or a neutron star from other dark bodies. It would be premature to suggest that extensive observational programs, beyond those already underway in the search for variable stars, be undertaken for the express purpose of discovering black hole eclipsing binary systems. We do, however, suggest that if our assumptions about the frequency of black hole binaries and the properties of their orbits and companions are close to the truth, then the probability of observing a black hole eclipse event, although small, is non-zero, and observational astronomers should be aware of the possibility of observing such an event. It should also be emphasized that, in the previous section, we have limited the discussion to systems in which the light amplification could exceed 5%, and includes possible cases in which the light amplification may have *very* large values.

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References

Adler, R., Bazin, M., Schiffer, M. 1965, Introduction to general relativity. McGraw-Hill, New York.
Cameron, A. G. W. 1971, *Nature* **229**, 178
Hawking, S. W. 1971, *Phys. Rev. Lett.* **26**, 1344.
Heintz, W. D. 1969, *J. R. astr. Soc. Can.* **63**, 275.
Oppenheimer, J., Snyder, H. 1939, *Phys. Rev.* **56**, 455.
Refsdal, Sjur 1964, *Mon. Not. R. astr. Soc.* **128**, 295.
Trimble, V. L., Thorne, K. S. 1969, *Ap. J.* **156**, 1013.
Zeldovich, Y. B., Novikov, I. D. 1964, *Soviet Phys. Dokl.* **9**, 246.

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