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Rest Mass in Special Relativity

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Specialized literature deals occasionally with potentials for which the rest mass of a particle is variable. Proper attention should be given in the teaching of special relativity to the relation between the rest mass and the potential field in all cases. The case of a scalar potential is considered.

Concerning the rest mass m of a particle we can always write[1]

$$dm/ds = X_\alpha (dx^\alpha/ds) \quad (\text{with } X_\alpha = dP_\alpha/ds), \quad (1)$$

which in the case of a scalar field, for instance, gives

$$dm/ds = X_\alpha (dx^\alpha/ds) = (\partial V/\partial x^\alpha)(dx^\alpha/ds) = dV/ds. \quad (2)$$

The constancy of the rest mass is not therefore a necessary consequence of special relativity, but implies an additional assumption concerning the field of force. Namely,

$$X_\alpha (dx^\alpha/ds) = 0. \quad (3)$$

Though this condition is satisfied in the case of a vector field[2], it is clearly not the case for a scalar field.

This is worth being stressed in view of the fact that in most textbooks on relativity,[3–8] the rest mass of a particle is considered to be constant. Moreover, there is a textbook[7] with a "proof" of the impossibility for the rest mass to be potential-dependent while another textbook uses a constant rest mass with a scalar potential.[8]

It should be stressed that we have to distinguish between the correct handling of the special relativity formalism (which in principle does not impose any *a priori* condition on the potential field except covariance) and the nature of the fields which may be found by some other theoretical or experimental considerations.

However, the importance of this point is not only formal. The specialized literature has and is dealing with cases in which the condition in Eq. (3) is not fulfilled. The scalar field of the hypothetical scalar meson has been investigated by Marx and Szamosi,[9] and Rosen.[10] Chevreton[11] and Rosen[12] have shown that the gravitational field of general relativity may be decomposed into a scalar field and a spin-2 field. Sexl[13] shows that within a large class of theories of gravitation, the scalar field is a component which plays an important role. Dicke[14] considers the existence of a

[1] J. L. Synge, *Relativity the Special Theory* (North-Holland Publ. Co., Amsterdam, 1956), p. 167. (Our convention is positive intervals, if they are timelike.)

[2] See Ref. 1, p. 395.

[3] C. Moller, *The Theory of Relativity* (Oxford University Press, London, 1962).

[4] R. C. Tolman, *Relativity Thermodynamics and Cosmology* (Oxford University Press, London, 1962).

[5] P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice–Hall, Inc., Englewood Cliffs, N. J., 1960).

[6] W. Pauli, *Theory of Relativity* (Pergamon Press, Ltd., London, 1958).

[7] J. Aharoni, *The Special Theory of Relativity* (Oxford University Press, London, 1959), p. 143. The author wrongly says that any potential dependence of the rest mass is incompatible with conservation of energy. The mistake consists in equating the work of the field with the assumed change in the particle energy. For instance, in the scalar case the particle energy is constant. This book is otherwise an excellent introduction to the applications of the theory of relativity in quantum theory.

[8] D. F. Lawden, *An Introduction to Tensor Calculus and Relativity* (Methuen and Company Ltd., London, 1962). He applies a canonical formalism to a scalar potential and assumes the rest mass to be constant. The expression obtained for the energy corresponds to that of a vector field. The formalism lacks covariance, and a correct relation between the expression of the energy and the nature of the potential cannot be obtained in this way. However, the monograph is a very good one. The need for clarifying the relation between rest mass and potential is the more evident.

[9] G. Marx and G. Szamosi, Bull. Acad. Polon. Sci. **III,** 475 (1954).

[10] N. Rosen, Bull. Res. Council Israel, Sec. A **6,** 55 (1956).

[11] M. Chevreton, C. R. Acad. Sci. Paris **259,** 304 (1964).

[12] N. Rosen, "Scale Transformations and Gravitons in General Relativity." (Unpublished report.)

[13] R. U. Sexl, Fortschr. Physik **15,** 269 (1967).

[14] R. H. Dicke, Phys. Rev. **126,** 1875 (1962).

cosmological scalar field. In nuclear physics all kinds of covariant interactions are considered *a priori* to be possible,[15] and when in some instances the scalar interaction is discarded, it is done in each particular case after the examination of the experimental evidence and its theoretical implications.[16] It is also to be noted that according to Kalman[17] the vector field is the only one for which the rest mass is constant; in particular, the rest mass for a tensor field is potential as well as velocity-dependent.

We therefore suggest that in the teaching of special relativity, proper attention should be given to the relation between the rest mass and the potential field.

It is instructive in this respect to compare the expression of the energy (the fourth component of momentum) in the case of a scalar and a vector field.

Starting from

$$dP^\alpha/ds=(d/ds)[m(dx^\alpha/ds)]=X^\alpha, \tag{4}$$

we have in all cases:

$$P^4=m\gamma=\int X^4ds. \quad \text{with} \quad \gamma=\left(1-\frac{v^2}{c^2}\right)^{-1/2}. \tag{5}$$

In the case of a time-independent scalar potential we have

$$X^4=-\partial V/\partial t=0. \tag{6}$$

Taking Eq. (5) into account we can write

$$P^4=m\gamma=W=\text{const.} \tag{7}$$

The energy of the particle is a constant of the motion.

Now, we have from Eq. (2)

$$m=V+m_0. \tag{8}$$

(m_0 being the rest mass in the absence of potential.) We may therefore write

$$P^4=W=(m_0+V)\gamma=\text{const.} \tag{9}$$

Equations (8) and (9) indicate that *in the case of a scalar potential, the potential energy contributes in toto to the rest mass.*

This may be contrasted with the result obtained for a static vector potential V whose only component different from zero is V_4. In this case we have[18]

$$P^4=W-V^4; \tag{10}$$

the energy of the particle is not constant.

For the constant total energy we may write

$$W=P^4+V^4=m\gamma+V^4. \tag{11}$$

Equations (9) and (11) suggest that in the scalar case the potential energy is localized in the frame of the moving particle, while in the vector case it is localized in the frame of the source of the field.

Taking Eq. (7) into consideration, Eq. (4) may be rewritten

$$X^\alpha=\frac{d}{ds}\left(m\gamma\frac{dx^\alpha}{dt}\right)=m\gamma\frac{d}{ds}\frac{dx^\alpha}{dt}=(m\gamma)\gamma a^\alpha$$

$$[\text{with } a^\alpha=(d^2x^\alpha/dt^2)], \tag{12}$$

or

$$X^\alpha=\gamma P^4a^\alpha. \tag{13}$$

If we define $f^\alpha=dP^\alpha/dt$, we have from Eq. (13)

$$f^\alpha=P^4a^\alpha \qquad (\text{with } P^4=\text{const}). \tag{14}$$

We obtain, therefore, for a scalar field the Newtonian relation between force and acceleration. However, the coefficient of proportionality is the constant total energy which includes the value of the rest mass, the potential, and the kinetic energy. *Force and acceleration are in this case parallel.*

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[15] W. S. C. Williams, *An Introduction to Elementary Particles* (Academic Press Inc., New York, 1961), p. 228. Incidentally, the neutral pion is believed to be a pseudoscalar particle. Its Yukawa potential, therefore, cannot be considered as the fourth component of a vector field.

[16] See Ref. 15, p. 263.

[17] G. Kalman, Phys. Rev. **123**, 384 (1961).

[18] W. Rindler, *Special Relativity* (Oliver and Boyd, Edinburgh, 1960), p. 100.