cent of the local mass density is in an unseen form, subject to the assumption that the unseen mass is strongly concentrated to the disk[9]. If the unseen mass has a more nearly spherical distribution, however, its mass can be very much greater than this[10].

For many years it has been a puzzle that giant elliptical galaxies have such a high mass-to-light ratio[11]. The present considerations lead us to suggest that the majority of the mass of such galaxies is in the form of collapsars.

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An Oscillating State as an Alternative to Gravitational Collapse

THE possibility of an oscillating universe is often disputed on the grounds that oscillations are not possible when there is no equilibrium position[1]. By the same argument, gravitational collapse can be considered as the only alternative when the mass is over critical (there is no configuration of equilibrium for such a big mass). The argument, however, cannot be generalized from the cosmological case to the case of a big mass, as we will see.

The existence of a critical mass of the order of that of the Sun follows from the equation of state obeyed by matter. This can be seen from the following example of a solution representing a mass as big as desired in equilibrium.

$$\mathrm{d}s^2 = \frac{-2}{1-ar^2}\,\mathrm{d}r^2 - r^2\,(\mathrm{d}\theta^2 + \sin^2 0\,\mathrm{d}\varphi^2) + ar^2\,\mathrm{d}t^2 \qquad (1)$$

from which we obtain

$$8\pi\rho = (1/2r^2) + (3a/2) \qquad (2)$$

$$8\pi p = (1/2r^2) - (3a/2) \qquad (3)$$

The equation of state is

$$\rho = p + k \text{ (with } k = 3a/8\pi) \qquad (4)$$

The element can be joined smoothly to the Schwarzschild exterior solution

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\right)^{-1} \mathrm{d}r^2 - r^2\,\mathrm{d}\,\Omega^2 + \left(1 - \frac{2m}{r}\right) \mathrm{d}t^2 \qquad (5)$$

giving for the gravitational mass the value

$$m = (1/3)\,(8\pi k)^{-1/2} \qquad (6)$$

and for the radius

$$R = (8\pi k)^{-1/2} \qquad (7)$$

Pressure and density become infinite at the origin, but there is a theorem[2] applicable in this case which ensures the existence of solutions with the same equation of state, and with a total mass as near as desired to that of our solution provided that the central pressure and density are high enough, though finite.

It is possible therefore to have as large a mass as desired in equilibrium provided we admit an equation of state of the form $\rho = p + k$ small enough. Oppenheimer and Volkoff's proof[3] of the existence of a critical mass of the order of that of the Sun relies on the known equation of state for cold neutron matter.

Misner and Zapolsky[4] have generalized the proof of ref. 3 for the case when matter with over nuclear density obeys an equation of state of the form $p = \rho(\gamma - 1)$ with $1 \leq \gamma \leq 2$ while for under nuclear densities the equation of state is the known equation for cold neutron matter.

Comparing my solution with that of Misner and Zapolsky, both propose the same equation of state for the core (if we take $\gamma = 2$). The essential difference is that the equation $\rho = p$ is considered by Misner and Zapolsky to be valid for over nuclear densities only, while in our case (for k very small) the equation $\rho = p$ remains approximately valid for a considerable range of under nuclear densities. When the equation $\rho = p$ can be extended to a quantity of matter sufficiently larger than that considered by Misner and Zapolsky, there may be equilibrium configurations for masses as great as desired.

The approach of Misner and Zapolsky is sound, so I do not propose my solution as a model for big masses in equilibrium. I accept therefore that there is no configuration of equilibrium for big masses of cold neutron matter. As such big masses collapse, however, most of the matter may become of over nuclear density.

The existence of restoring forces must now be studied with due regard to the equation of state prevailing in the dynamical process during which there exists a much higher ratio of matter with over nuclear density to matter with under nuclear densities than in the static case.

Misner and Zapolsky did not calculate the possibility of equilibrium for such higher ratios of matter obeying the $\rho = p$ equation to matter not obeying it (such higher ratios are unrealistic in the static case). My example proves that in this case there is an equilibrium configuration and therefore there may be restoring forces to this position of equilibrium.

An over critical mass could therefore oscillate in the following way: (1) It starts collapsing because it cannot remain in equilibrium. (2) As it collapses, the ratio of mass with over nuclear to mass with under nuclear density increases and the equation of state of that part of the body, the density of which goes from under nuclear density to over nuclear density, does change. As a result, restoring forces build up. (3) Under the effect of the restoring forces the collapse stops, to be followed by the expansion of the body. (4) While expanding, the ratio of over nuclear densities to under nuclear densities decreases. The body therefore does not cross a position of equilibrium but completes the cycle by reaching the initial state for which no equilibrium position exists.

It would be interesting to establish the precise ratio (as a function of total mass) which separates the cases for which there is or there is not restoring forces. It would be helpful to calculate the limiting mass for which the restoring forces are reversing the collapsing movement before the crossing of the Schwarzschild radius. When the Schwarzschild radius is to be crossed (from the point of view of a comoving observer), the body is to be considered as a collapsing non-oscillating one from the point of view of an external observer who will observe the body as tending asymptotically towards the Schwarzschild radius.

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Long Term Variations of Pulsar Intensities

Pulsar intensity variations may be divided roughly into three classes. There are rapid variations on a time-scale of seconds to minutes which occur simultaneously over a wide range of frequency and must therefore be intrinsic to the source[1]. On a somewhat longer time-scale there are variations typically over a few minutes to a few hours which correlate only over limited bandwidths. There is strong evidence that they are caused by irregular diffraction in the interstellar medium[2,3]. Finally, there are variations on a time-scale of days to months[4] about which little is known because extended regular observations are required. In this report we present some new results obtained during timing observations carried out on a routine basis over 11 months.

The five pulsars CP 0808, CP 0834, CP 0950, CP 1133 and CP 1919 were observed during meridian transit with the 81·5 MHz phased array at Cambridge. A daily measure of the mean intensity was obtained by averaging the ten largest pulses to occur during the 4 min or so that the source was in the beam. Considerable day to day variations must arise from the interstellar fluctuations (extrapolating from the data of Rickett[3]), so the daily values were smoothed by taking a 7 day running mean.

The results are shown in Fig. 1, together with some isolated values at 408 MHz. All five pulsars show similar behaviour in that variations on time-scales extending from a few days to several weeks are clearly evident. CP 0950 is, possibly, the most variable source and has shown the greatest extreme fluctuations. This is not fully apparent in Fig. 1, because the timing records from which the data were taken were saturating for very large pulse intensities. It should be noted, however, that in November 1967, CP 0950 exhibited an increase by a factor of about 200 (ref. 5). The variability of this pulsar is also shown by the data of Downs *et al.* at 12·5 cm (ref. 6).

The points at 408 MHz were included in an attempt to investigate the correlation of long period variations at another frequency. We thank the Nuffield Radio Astronomy Laboratories, Jodrell Bank, for these data, which were obtained from observations with the Mark I telescope usually tracking each source for about an hour. Because the medium period fluctuations are slower at 408 MHz, these points were smoothed by taking a mean over 4 days wherever possible. There is no clear correlation between the strength at the two frequencies for either source, but for CP 0950 the peaks and troughs at 81·5 MHz tend to coincide with those at 408 MHz. The correlation coefficient for the CP 0950 data (unsmoothed at 408 MHz) is 0·37, which is significant at the 5 per cent confidence level. The correlation is almost unchanged by smoothing. There is no significant correlation for the CP 1919 data. This indicates that there may be some correlation of long-term variations over this frequency range, but more extended observations are needed to settle this point.

There seems to be no possibility of accounting for long term variations of intensity by random diffraction in the interstellar medium. Irregularities in the interstellar medium produce a diffraction pattern at the Earth which appears as an intensity variation in time due to the transverse velocities of the irregularities relative to the

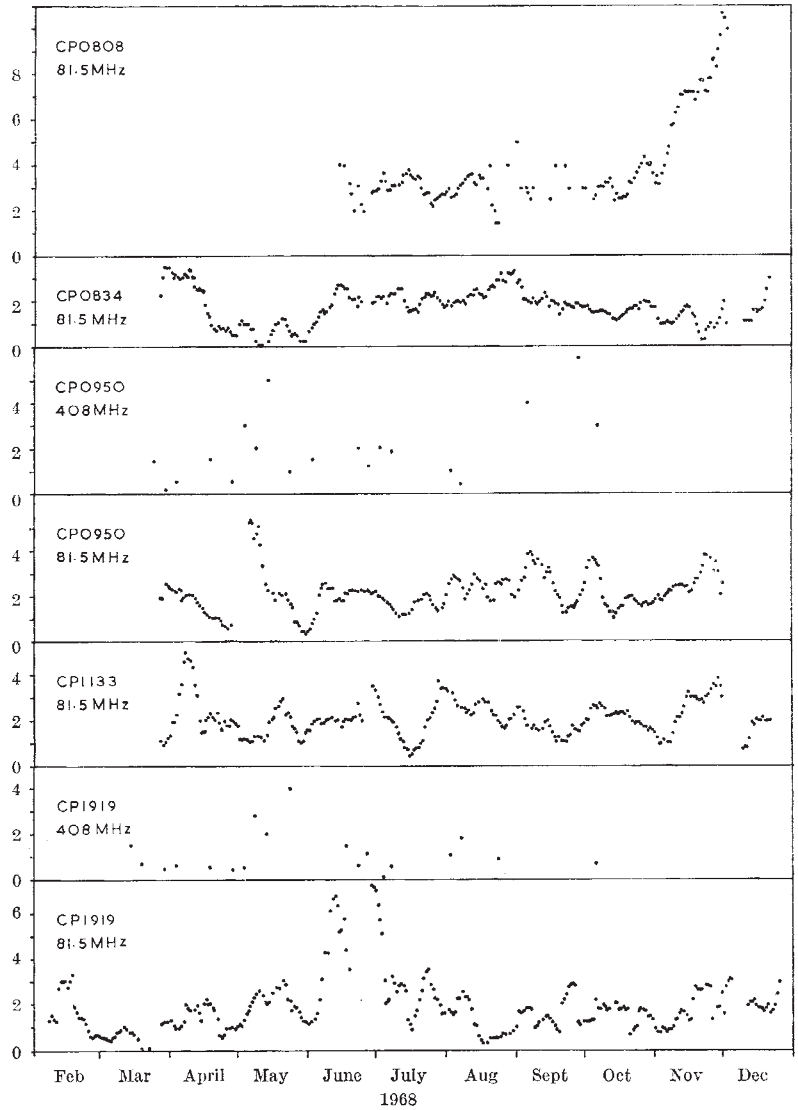

Fig. 1. Plots of intensities (7 day running mean) for five pulsars with an arbitrary linear intensity scale.